Fusing the Navigation Information of Dual Foot-Mounted Zero-Velocity-Update-Aided Inertial Navigation Systems

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***Abstract*—A *range constraint* method viz. *centroid method* is proposed to fuse the navigation information of dual (right and left) foot-mounted Zero-velocity-UPdaTe (ZUPT)-aided Inertial Navigation Systems (INSs). Here, the *range constraint* means that the distance of separation between the position estimates of right and left foot ZUPT-aided INSs cannot be greater than a quantity known as *foot-to-foot maximum separation*. We present the experimental results which illustrate the applicability of the proposed method. The results show that the proposed method significantly enhances the accuracy of the navigation solution when compared to using two uncoupled foot-mounted ZUPT-aided INSs. Also, we compare the performance of the proposed method with the existing data fusion methods.**

Keywords-Inertial navigation, range constraint, Zero-velocity detection, Kalman filter.

I. Introduction

A robust, accurate, and infrastructure-free positioning system with seamless outdoor and indoor coverage is a highly needed tool for increasing the safety in emergency response and military urban operations [1]. Global Positioning System (GPS) receiver is commonly used tool for outdoor navigation. Unfortunately, GPS receiver is often unable to provide the accuracy and the availability in indoor environments where pedestrian navigation is commonly required. Hence, the main technical challenge is to create a sufficiently accurate positioning system in all kinds of indoor environments. There are wide range of applications where indoor navigation is useful. For example, tracking the location of first responders in an harsh environment or location of customers in a shopping mall for targeted advertising [2].

An alternative to the GPS is the low-cost inertial sensor based *dead reckoning* system known as *inertial navigation system*. Here, *dead reckoning* is the process of estimating the current position of an object by keeping track of its movements relative to a known starting point [3]. INS takes in inertial sensor measurements as input and outputs position, velocity and attitude estimates by executing *navigation equations*. Inertial sensor or Inertial Measurement Unit (IMU) is the main component of INS and it works by detecting the current rate of linear accleration using 3-axis accelerometers and detects the changes in rotational attributes (roll, pitch and yaw) using 3-axis gyroscopes. The equations integrating the gyroscope and the accelerometer measurements to estimate the *navigation states* (position, velocity and attitude angles) is called *navigation equations* [4].

As pointed out in [5], one of the drawback of low-cost inertial sensor based INS is the unbounded position and velocity error growth. A technique to bound the error growth is using *zero-velocity update* as explained in [6]. An open source project which implements embedded foot-mounted ZUPT-aided INS is OpenShoe [7]. Refer http://www.openshoe.org/ or [7] for implementation details of OpenShoe navigation system.

But as pointed out in [8], one of the drawbacks of the of the existing foot-mounted ZUPT-aided INS is the *systematic heading drift*. The estimated trajectories drift away from the actual path as time progresses. Another important observation to be made is that the drifts obtained are symmetrical. These errors are large scale manifestations of modeling errors in the system. One possible way these errors can be mitigated is by using foot-mounted INS on both feet as suggested in [9]-[12] such that the symmetrical modeling errors cancel out.

In this report, we propose a method to fuse the navigation information or navigation states of dual foot-mounted ZUPT-aided INSs. The proposed method is based on the intuition that the distance of separation between the position estimates of right and left foot INSs cannot be greater than a quantity known as *foot-to-foot maximum separation* (γ). Based on this intuition, we constrain the position estimates of right and left foot-mounted ZUPT-aided INSs, hence the method is called *range constraint* method.

Outline of the paper: In Section II, we describe the *centroid method* problem formulation and solution. In Section III, we describe the *centroid method* algorithm. In Section IV, we present the experimental results. In Section V, conclusions are presented.

II. Centroid Method

In this section, we will describe how the range constraint can be used to fuse the navigation information of right and left foot-mounted ZUPT-aided INSs.

A. Problem formulation

Let $\hat{\mathbf{x}}_k^{(\mathrm{R})} \in \mathbb{R}^9$ be the navigation state vector of right foot-mounted ZUPT-aided INS at time instant $k \in \mathbb{N}^+$and is defined as:

$$\hat{\mathbf{x}}_k^{(R)} \triangleq \begin{bmatrix} \hat{\mathbf{p}}_k^{(R)} & \hat{\mathbf{v}}_k^{(R)} & \hat{\boldsymbol{\theta}}_k^{(R)} \end{bmatrix}^T \tag{1}$$

where $\hat{\mathbf{p}}_k^{(R)} \in \mathbb{R}^3$, $\hat{\mathbf{v}}_k^{(R)} \in \mathbb{R}^3$ and $\hat{\boldsymbol{\theta}}_k^{(R)} \in \mathbb{R}^3$ are respectively the position, velocity, and attitude (roll, pitch and yaw angles) estimates of right foot-mounted ZUPT-aided INS at time instant $k \in \mathbb{N}^+$.
Similarly, let $\hat{\mathbf{x}}_k^{(\mathrm{L})} \in \mathbb{R}^9$ be the navigation state vector of left foot-mounted ZUPT-aided INS at time instant $k \in \mathbb{N}^+$and is defined as:

$$\hat{\mathbf{x}}_k^{(L)} \triangleq \begin{bmatrix} \hat{\mathbf{p}}_k^{(L)} & \hat{\mathbf{v}}_k^{(L)} & \hat{\boldsymbol{\theta}}_k^{(L)} \end{bmatrix}^T \tag{2}$$

where $\hat{\mathbf{p}}_k^{(L)} \in \mathbb{R}^3$, $\hat{\mathbf{v}}_k^{(L)} \in \mathbb{R}^3$ and $\hat{\boldsymbol{\theta}}_k^{(L)} \in \mathbb{R}^3$ are respectively the position, velocity, and attitude estimates of left foot-mounted ZUPT-aided INS at time instant $k \in \mathbb{N}^+$.
Next, define the joint position estimate vector

$$\hat{\mathbf{p}}_k \triangleq \left[\left(\hat{\mathbf{p}}_k^{(R)}\right)^T \quad \left(\hat{\mathbf{p}}_k^{(L)}\right)^T \right]^T \tag{3}$$

Define the matrix

$$\mathbf{L} \triangleq \begin{bmatrix} \mathbf{I}_3 & -\mathbf{I}_3 \end{bmatrix} \tag{4}$$

where $\mathbf{I}_q$ is the identity matrix of size q.

It is intuitive that at any instant of time $k \in \mathbb{N}^+$, the distance between the position estimates of right and left foot-monted ZUPT-aided INSs cannot be greater than γ; i.e., the following condition must hold:

$$||\mathbf{L}\hat{\mathbf{p}}_k||_2 \; = \; \left|\left|\hat{\mathbf{p}}_k^{(R)} \; - \; \hat{\mathbf{p}}_k^{(L)}\right|\right|_2 \; \leq \; \gamma, \;\; \forall \, k \in \mathbb{N}^+$$

In the Constrained Least Squares (CLS) framework, the range constraint problem is formulated as:

$$\mathbf{p}_k = \underset{\mathbf{p} \, \in \, \mathbb{R}^6}{\text{argmin}} \; \left(||\hat{\mathbf{p}}_k - \mathbf{p}||_2^2 \right) \quad \text{s.t.} \quad ||\mathbf{L}\mathbf{p}_k||_2^2 \leq \gamma^2 \tag{5}$$

where $\mathbf{p}_k$ is the solution of the constrained least squares problem and is defined as:

$$\mathbf{p}_k \triangleq \left[\left(\mathbf{p}_k^{(R)}\right)^T \quad \left(\mathbf{p}_k^{(L)}\right)^T \right]^T \tag{6}$$

B. Solution

Lagrange function for (5) with Lagrange multiplier λ is given by [13]:

$$J\left(\mathbf{p}_k, \lambda\right) \;\; \triangleq \;\; ||\hat{\mathbf{p}}_k - \mathbf{p}_k||_2^2 \;\; + \;\; \lambda \, \psi\left(\mathbf{p}_k\right) \tag{7}$$

where

$$\psi\left(\mathbf{p}_k\right) \;\; = \;\; ||\mathbf{L}\mathbf{p}_k||_2^2 \; - \; \gamma^2 \tag{8}$$

The *Lagrangian condition* yields:

$$\frac{\partial J\left(\mathbf{p}_k, \lambda\right)}{\partial \mathbf{p}_k} \; = \; \mathbf{0} \;\; \Longrightarrow \;\; \left(\mathbf{I}_6 \; + \; \lambda \, \mathbf{L}^T\mathbf{L}\right) \, \mathbf{p}_k \; = \; \hat{\mathbf{p}}_k \tag{9}$$

$$\frac{\partial J\left(\mathbf{p}_k, \lambda\right)}{\partial \lambda} \; = \; 0 \quad \Longrightarrow \quad \psi\left(\mathbf{p}_k\right) \; = \; 0 \tag{10}$$

The solution of (9) is

$$\mathbf{p}_k \; = \; \left(\mathbf{I}_6 \; + \; \lambda \, \mathbf{L}^T\mathbf{L}\right)^{-1} \, \hat{\mathbf{p}}_k \tag{11}$$

Solve for λ:
First note that

$$\left(\mathbf{I}_6 + \lambda \, \mathbf{L}^T\mathbf{L}\right)^{-1} = \frac{1}{1+2\lambda} \begin{bmatrix} (1+\lambda)\,\mathbf{I}_3 & \lambda\mathbf{I}_3 \\ \lambda\mathbf{I}_3 & (1+\lambda)\,\mathbf{I}_3 \end{bmatrix} \tag{12}$$

provided $\lambda \neq -0.5$.
Substitute (11) in (10)

$$\Longrightarrow \; \left|\left|L\left(\mathbf{I}_6 \; + \; \lambda \, \mathbf{L}^T\mathbf{L}\right)^{-1} \, \hat{\mathbf{p}}_k\right|\right|_2^2 \; = \; \gamma^2$$

Substitute (12) and simplifying

$$\Longrightarrow \; \frac{1}{\left(1+2\lambda\right)^2} \; ||\mathbf{L} \, \hat{\mathbf{p}}_k||_2^2 \; = \; \gamma^2$$

Solving for $\lambda \geq 0$

$$\lambda \; = \; 0.5 \, \left(\frac{||\mathbf{L} \, \hat{\mathbf{p}}_k||_2}{\gamma} \; - \; 1 \right) \tag{13}$$

Note that

$$||\mathbf{L} \, \hat{\mathbf{p}}_k||_2 \; = \; \left|\left|\hat{\mathbf{p}}_k^{(R)} \; - \; \hat{\mathbf{p}}_k^{(L)}\right|\right|_2 \; \triangleq \; \hat{d}_k \tag{14}$$

Substitute (14) in (13) implies

$$\lambda \; = \; 0.5 \, \left(\frac{\hat{d}_k}{\gamma} \; - \; 1 \right) \tag{15}$$

Substitute (12) in (11) and simplifying:

$$\mathbf{p}_k^{(R)} \; = \; \frac{1+\lambda}{1+2\,\lambda} \, \hat{\mathbf{p}}_k^{(R)} \; + \; \frac{\lambda}{1+2\,\lambda} \, \hat{\mathbf{p}}_k^{(L)} \tag{16}$$

$$\mathbf{p}_k^{(L)} \; = \; \frac{\lambda}{1+2\,\lambda} \, \hat{\mathbf{p}}_k^{(R)} \; + \; \frac{1+\lambda}{1+2\,\lambda} \, \hat{\mathbf{p}}_k^{(L)} \tag{17}$$

Substitute (15) in (16) and (17), implies

$$\mathbf{p}_k^{(R)} \; = \; \frac{\left(\hat{d}_k \, + \, \gamma\right) \, \hat{\mathbf{p}}_k^{(R)} \; + \; \left(\hat{d}_k \, - \, \gamma\right) \, \hat{\mathbf{p}}_k^{(L)}}{2 \, \hat{d}_k} \tag{18}$$

$$\mathbf{p}_k^{(L)} \; = \; \frac{\left(\hat{d}_k \, - \, \gamma\right) \, \hat{\mathbf{p}}_k^{(R)} \; + \; \left(\hat{d}_k \, + \, \gamma\right) \, \hat{\mathbf{p}}_k^{(L)}}{2 \, \hat{d}_k} \tag{19}$$

III. Range Constrained Right and Left Foot-mounted ZUPT-aided INSs

In this section, we will describe the algorithm which applies the *centroid method* in Kalman filter [14] framework to fuse the navigation information of dual (right and left) foot-mounted ZUPT-aided INSs.

A. Inertial Navigation System

IMU data (accelerometer and gyroscope data) is processed to compute the *navigation states* (position, velocity and attitude angles) [4], [15] and [16].

$$\hat{\mathbf{x}}_k^{(i)} \; = \; f\left(\hat{\mathbf{x}}_{k-1}^{(i)}, \, \tilde{\mathbf{s}}_k^{(i)}, \, \tilde{\boldsymbol{\omega}}_k^{(i)}\right) \tag{20}$$

where $\hat{\mathbf{x}}_0^{(i)}$ is the initial navigation state and $i \in \{R, L\}$ denotes either right foot INS or left foot INS.
Time dynamics of the errors in navigation state $\left(\delta\hat{\mathbf{x}}_k^{(i)} \in \mathbb{R}^9\right)$ is described by the state-space model

$$\delta\hat{\mathbf{x}}_k^{(i)} \; = \; \mathbf{F}_k^{(i)} \, \delta\hat{\mathbf{x}}_{k-1}^{(i)} \; + \; \mathbf{G}_k^{(i)} \, \mathbf{w}_k^{(i)} \tag{21}$$

where $\mathbf{F}_k^{(i)}$ and $\mathbf{G}_k^{(i)}$ denote the state transition and process noise gain matrix, respectively. $\mathbf{w}_k^{(i)} \in \mathbb{R}^6$ denotes the perturbation in IMU measurement, which is assumed white and to have the covariance matrix $\mathbf{Q}^{(i)}$. Hence, state covariance matrix is described by

$$\mathbf{P}_k^{(i)} \; = \; \mathbf{F}_k^{(i)}\mathbf{P}_{k-1}^{(i)}\left(\mathbf{F}_k^{(i)}\right)^T \; + \; \mathbf{G}_k^{(i)}\mathbf{Q}^{(i)}\left(\mathbf{G}_k^{(i)}\right)^T \tag{22}$$

B. Zero-velocity Update

We will use the zero-velocity detector viz. *SHOE detector* [17] to detect the zero-velocity conditions. If ZUPT is **on**, then using zero-velocity as the pseudo-measurement in Kalman filter framework, we correct the navigation states as explained in [3] and [6]. The steps involved in applying ZUPT are given below. $i \in \{R, L\}$

1) Compute the Kalman gain:

$$\mathbf{K}_k^{(i)} \; = \mathbf{P}_k^{(i)}\left(\mathbf{H}_{vel}\right)^T\left[\mathbf{H}_{vel}\mathbf{P}_k^{(i)}\left(\mathbf{H}_{vel}\right)^T \; + \; \mathbf{R}_{vel}\right]^{-1}$$

where $\mathbf{H}_{vel} \; = \; \left[\mathbf{0}_{3\times3} \quad \mathbf{I}_{3\times3} \quad \mathbf{0}_{3\times3}\right]$ is the velocity pseudo-measurement observation matrix and $\mathbf{R}_{vel}$ is the velocity pseudo-measurement noise covariance matrix.

2) Correct the navigation state vector using the velocity pseudo-measurement:

$$\hat{\mathbf{x}}_k^{(i)} \;\; = \;\; \hat{\mathbf{x}}_k^{(i)} \;\; + \;\; \mathbf{K}_k^{(i)} \; \left[\; \mathbf{v}_k^{(i)} \;\; - \;\; \mathbf{H}_{vel} \, \hat{\mathbf{x}}_k^{(i)}\right]$$

where $\mathbf{v}_k^{(i)} \; = \; [0 \;\; 0 \;\; 0]^T$ indicates velocity is zero.

3) Correct the state covariance matrix:

$$\mathbf{P}_k^{(i)} \;\; = \;\; \left[\mathbf{I}_{9\times9} \;\; - \;\; \mathbf{K}_k \, \mathbf{H}_{vel}\right] \; \mathbf{P}_k^{(i)}$$

C. Range Update

If the distance between the position estimates of right and left foot-mounted ZUPT-aided INSs is greater than γ (foot-to-foot maximum separation), then range constrained position estimates of right and left foot is computed as explained in Section II. Using range constrained position estimates as the pseudo-measurement in Kalman filter framework, we correct the navigation states. The steps involved in applying *Range UPdaTe* (RUPT) are given below. $i \in \{R, L\}$

1) Compute the Kalman gain:

$$\mathbf{K}_k^{(i)} = \mathbf{P}_k^{(i)}\left(\mathbf{H}_{pos}\right)^T\left[\mathbf{H}_{pos}\mathbf{P}_k^{(i)}\left(\mathbf{H}_{pos}\right)^T \; + \; \mathbf{R}_{pos}\right]^{-1}$$

where $\mathbf{H}_{pos} \; = \; \left[\mathbf{I}_{3\times3} \quad \mathbf{0}_{3\times3} \quad \mathbf{0}_{3\times3}\right]$ is the position pseudo-measurement observation matrix and $\mathbf{R}_{pos}$ is the position pseudo-measurement noise covariance matrix.

2) Correct the navigation state vector using the position pseudo-measurement:

$$\hat{\mathbf{x}}_k^{(i)} \;\; = \;\; \hat{\mathbf{x}}_k^{(i)} \;\; + \;\; \mathbf{K}_k^{(i)} \; \left[\; \mathbf{p}_k^{(i)} \;\; - \;\; \mathbf{H}_{pos} \, \hat{\mathbf{x}}_k^{(i)}\right]$$

where $\mathbf{p}_k^{(i)} \; \in \; \mathbb{R}^3$ is the range constrained position estimate of i-th system.

3) Correct the state covariance matrix:

$$\mathbf{P}_k^{(i)} \;\; = \;\; \left[\mathbf{I}_{9\times9} \;\; - \;\; \mathbf{K}_k \, \mathbf{H}_{pos}\right] \; \mathbf{P}_k^{(i)}$$

Pseudo code for the algorithm which applies *centroid method* in Kalman filter framework to fuse the navigation information of dual (right and left) foot-mounted ZUPT-aided INSs is given in Algorithm 1.

IV. Experiment and Results

The proposed algorithm is tested by conducting the following experiment. A user, equipped with one OpenShoe unit on each foot walked different paths on the corridor of the first floor of Signal Processing (SP) building, Department of ECE, Indian Institute of Science (IISc), Bangalore, India. The corridor is in 'U' shape with sharp 90° turns. The corridor length is 34.8 [m] and the width is 23.4 [m]. Right and left foot OpenShoe unit data (inertial data) corresponding to straight line path, inverted 'L' and 'U' path were recorded. The inertial data collected from right and left OpenShoe units was processed with Algorithm 1. By setting foot-to-foot maximum separation to infinity ($\gamma \; = \; \infty \, [m]$), we get unconstrained system and by setting $\gamma = 1 \, [m]$, we get range constrained system. The estimated trajectories for straight line, inverted 'L' and inverted 'U' path are shown in Figure 1. We observe that constrained system has improved the accuracy of the navigation solution.
To compare the performance of the proposed algorithm with the existing algorithms in [9] and [18], we make use of the datasets used in [9] (datsets is available in

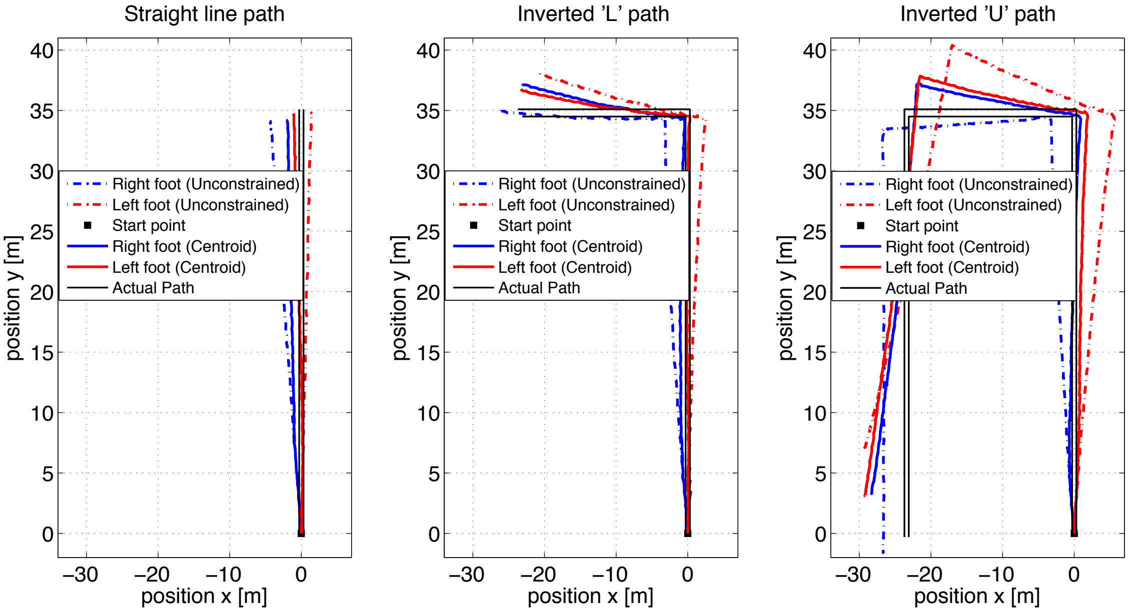

Figure 1: Trajectory comparison of unconstrained and range constrained (*centroid method*) system.

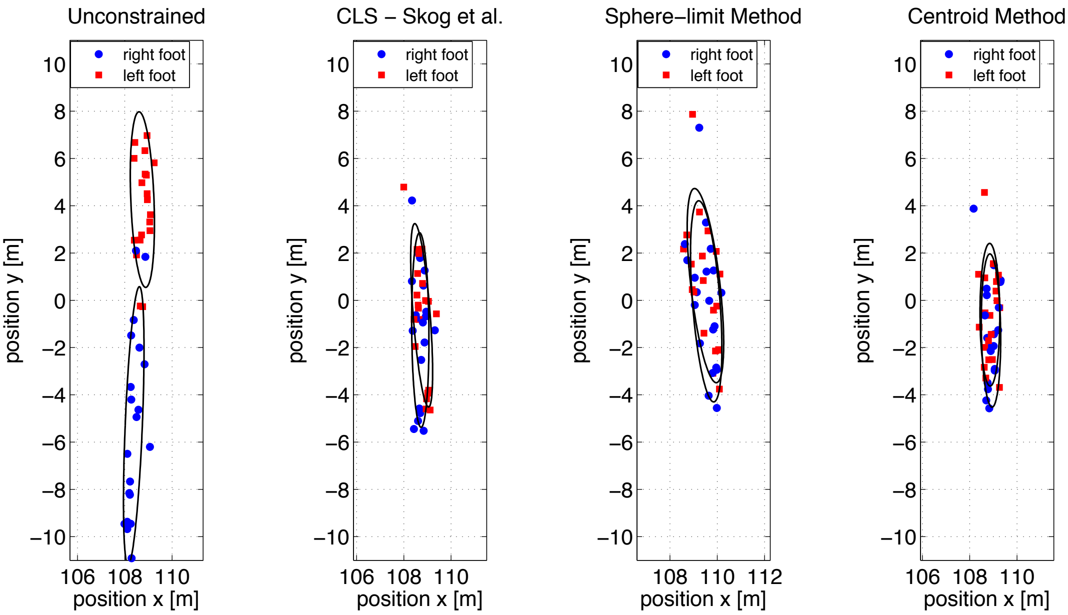

Figure 2: Scatter plot of end position for unconstrained system, *CLS method* constrained system, *sphere-limit method* constrained system, and *centroid method* constrained system.

http://www.openshoe.org/). A user, equipped with one OpenShoe unit on each foot, wlaked 110 metres on level ground along a straight line at a normal gait speed. A total of 40 datasets were collected. Figure 2 shows the end position of the right and left foot for unconstrained, *CLS method* [9], *sphere-limit method* [18] and *centroid method* algo-

Algorithm 1 Pseudo code for the proposed range constraint algorithm.

$k \leftarrow 0$, $c_z \leftarrow -\tau_z$
$\hat{\mathbf{x}}_k^{(R)} \leftarrow$ **Process**{Initial navigation state of right-INS}
$\hat{\mathbf{x}}_k^{(L)} \leftarrow$ **Process**{Initial navigation state of left-INS}
$\mathbf{P}_k^{(R)} \leftarrow$ **Process**{Initial covariance matrix of right-INS}
$\mathbf{P}_k^{(L)} \leftarrow$ **Process**{Initial covariance matrix of left-INS}
loop
    $k \leftarrow k + 1$
    $\hat{\mathbf{x}}_k^{(R)} \leftarrow$ **Process**{Right-INS *navigation equations*}
    $\hat{\mathbf{x}}_k^{(L)} \leftarrow$ **Process**{Left-INS *navigation equations*}
    $\mathbf{P}_k^{(R)} \leftarrow$ **Process**{Update right covariance matrix}
    $\mathbf{P}_k^{(L)} \leftarrow$ **Process**{Update left covariance matrix}
    $Z_k^{(R)} \leftarrow$ **Process**{Right-INS zero-velocity detector}
    $Z_k^{(L)} \leftarrow$ **Process**{Left-INS zero-velocity detector}
    if $Z_k^{(R)}$ is **on then**
        $\{\hat{\mathbf{x}}_k^{(R)}, \mathbf{P}_k^{(R)}\} \leftarrow$ **Process**{Right-INS ZUPT}
    end if
    if $Z_k^{(L)}$ is **on then**
        $\{\hat{\mathbf{x}}_k^{(L)}, \mathbf{P}_k^{(L)}\} \leftarrow$ **Process**{Left-INS ZUPT}
    end if
    if $||\mathbf{L}\hat{\mathbf{p}}_k||_2 > \gamma$ **and** $k - c_z > \tau_z$ **then**
        $\{\hat{\mathbf{x}}_k^{(R)}, \mathbf{P}_k^{(R)}\} \leftarrow$ **Process**{Right-INS RUPT}
        $\{\hat{\mathbf{x}}_k^{(L)}, \mathbf{P}_k^{(L)}\} \leftarrow$ **Process**{Left-INS RUPT}
        $c_z \leftarrow k$
    end if
end loop

rithms. Applying the range constraints can be seen to have significantly improved the accuracy of the navigation solution when compared to using two uncoupled foot-mounted ZUPT-aided INSs and the results are comparable with the existing data fusion methods.

V. Conclusions

We have proposed a *range constraint* method viz. *centroid method* to fuse the navigation information of right and left foot-mounted ZUPT-aided INSs. The range constrained solution is incorporated in a Kalman filter framework. The use of the proposed method has been shown to enhance the accuracy of the navigation solution when compared to using two uncoupled foot-mounted ZUPT-aided INSs. We also presented the experimental results which illustrate the applicability of the proposed method. The performance of the proposed method is compared with the existing data fusion methods. In the future, we plan to study the analytic measure to compare the performance of different data fusion algorithms.

Acknowledgements

This project is jointly funded by Department of Science and Technology, Govt. of India and VINNOVA, Sweden. We would also like to thank Dr. Isaac Skog and John-Olof Nilsson for making the datasets and code readily available at http://www.openshoe.org/

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